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| **Routh-hurwitz stability criterion:** Tests whether a system is stable (aka does not increase to infinity, all roots in LHP). From the transfer function if there is a sign change in the denominator, then then the system is unstable. Then create the Hurwitz matrix: where and . If a first column term is 0 but the remaining terms aren’t 0 or there are no remaining terms, then the 0 is replaced by a very small positive number ϵ and the rest of the array is evaluated. If all coefficients in a row are 0, then take the derivative of the auxiliary polynomial from the previous row. For example, take the derivative of the s4 row, , and use that: . The number of poles in the RHP is equal to the number of sign changes in the first column of the complete matrix. |

**Second order systems**

|  |  |  |  |  |  |  |  |  |  |
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|  | | | **Undamped natural frequency** | | **Damping ratio** | | | **Damped natural frequency** | |
|  | | |  | |  | | | (imaginary)  (real) | |
| **Maximum overshoot (unity)** | | | **Maximum overshoot (not unity)** | | **Maximum overshoot percentage** | | | **Pole locations** | |
| or | | |  | |  | | |  | |
| **Settling time** | | | **Rise/Peak time** | | **Damping** | | | **Chart** | |
| (2%)  (5%) | | |  | | : undamped  : underdamped  : critically-damped  : overdamped | | |  | |
| **Laplace transform pairs** | |  | |  | |  | **Properties of Laplace transforms** | |  |
|  |  |  | |  | |  |  | |  |
| Unit impulse |  |  | |  | |  |  | |  |
| Unit step |  |  | |  | |  |  | |  |
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|  |  |  | |  | |  | if exists | |  |
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|  |  |  | |  | |  | if exists | |  |
|  |  |  | |  | |  |  | |  |
|  |  |  | |  | |  | **Initial value theorem** | |  |
|  |  |  | |  | |  | **Final value theorem** | |  |

**Steady state error**

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**Steady state error in terms of gain K**

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| --- | --- | --- | --- | --- |
| # of poles at s=0 | Step input | Ramp input | Acceleration input |  |
| Type 0 system |  |  |  | System is of type N for a system with open-loop transfer function , representing a pole of multiplicity N at the origin. |
| Type 1 system | 0 |  |  |
| Type 2 system | 0 |  |  |

**Root-locus plots**

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| --- | --- |
| **System requirements:**  - Damping ratio: roots must be within cone “>”  - Time for exponential decay to half: roots must be to the left of line “|”  - Natural frequency: roots must be inside lines “=” | **Equations:**  **Characteristic equation:** where is the open-loop transfer function, *H(s)* is the feedback. For unity feedback, the equation is . To get the characteristic equation, group all *K* terms and divide by non-*K* terms.  **Magnitude condition:**  **Angle condition:**  **Angles of asymptotes:** where *n* is the number of poles, *m* is the number of zeros and .  **Centroid of asymptotes:**  **Breakaway/break-in points:** Rearrange the characteristic equation so that *K* is equal to a polynomial. Then the breakaway/break-in points are the values of *s* where the derivative of that polynomial equals 0. If a real root is in the root locus, then it is a break point. If a real root is not in the root locus, then it is not. If two roots are a complex conjugate pair, check which gives a possible value of *K* (usually ).  **Angle of departure from complex pole:** 180 – sum of angles from other poles + sum of angles from zeros  **Angle of arrival at a complex zero:** 180 – sum of angles from other zeros + sum of angles from poles  **Root loci cross imaginary axis:** Create the Routh array from the characteristic equation. Then find K such that the value of the first column of the term is 0. Then use that value to find the roots of the auxiliary equation of the row. The crossing points are the roots and the gain is K.  **Gain (*K*) at point *s* on root locus:** Substitute *s* into the characteristic equation and solve for *K*. |
| **Drawing root-locus:**  **1.** There are n lines (loci) where n is the degree of *Q* or *P*, whichever is greater.  **2.** As K increases from 0 to infinity, the roots move from the poles of to the zeros of .  **3.** When roots are complex they occur in conjugate pairs.  **4.** At no time will the same root cross over its path.  **5.** The portion of the real axis to the left of an odd number of open loop poles and zeros are part of the loci.  **6.** Lines leave and enter the real axis at 90°.  **7.** If there are not enough poles or zeros to make a pair then the extra lines go to or come from infinity.  **8.** Lines go to infinity along asymptotes.  **9.** If at least 2 lines go to infinity, then the sum of all roots is constant.  **10.** K going from 0 to negative infinity can be drawn by reversing rule 5. and adding 180° to the asymptote angles. |
| **Dominant poles:** Poles near the imaginary axis have a much larger effect on the system and so are dominant. A zero near a corresponding pole reduces the pole’s effect. |
| **Lead compensator:** zero is closer to origin than pole  - Moves asymptotes, closed-loop poles to the left. Improves stability.  - If a point is on the root locus (use only 2 dominant poles):  - Place zero just to the left of the last open loop pole  - Then calculate position of pole to satisfy the sum.  - Use magnitude condition to calculate gain K.  - The transfer function of the compensator is then | **Lag compensator:** pole is closer to origin than 0  - Moves asymptotes, closed-loop poles to the right. Reduces steady state error.  - Find the static error constant:  - The ratio of the compensator’s zero to pole should be where *Kcomp* is the desired error constant and *Kerror* is the uncompensated error constant.  - Arbitrarily select a small *zc*, then solve the equation to find *pc*.  - The transfer function of the compensator is then . |

**Bode plots**

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| Steady state phase and gain can be calculated from a transfer function by setting *s* to . The result is a complex number where the magnitude of the result is the gain, and the angle is the phase. Create a plot of magnitude (in dB) vs frequency (log scale) and a plot of phase (in degrees) vs frequency (log scale). | |
| Complex transfer functions: Add together magnitude and phase responses of each part of the TF. | |
| Gain and phase of a zero is the reflection of the gain of a pole about the horizontal axis. | |
| Magnitude into dB: | |
| If , magnitude is *K* and phase is 0 if and -180 if . Magnitude is constant (straight line). Phase is constant (straight line). | |
| If , magnitude is and phase is -90. Magnitude is linear (-20 dB/decade). Phase is constant (straight line). | |
| If .  Magnitude: When , and when . So magnitude is a straight line until a, and then a sloped line (-20 dB/decade).  Phase: When , when . Then connect those lines with a sloped straight line (-45 degrees/decade). | |
| **Nyquist plots**    *Z* is the number of zeros in the RHP  *N* is the number of clockwise encirclements around -1 (negative if counterclockwise).  *P* is the number of open loop RHP poles  The closed-loop system is stable if | A picture containing diagram  Description automatically generated |

Line chart

Description automatically generated with medium confidence